SINGAPORE MATH: CAN IT HELP CLOSE THE U.S. MATHEMATICS LEARNING GAP?

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On the 2003 Trends in International Mathematics and Science Study (TIMSS; Mullis, Martin, Gonzalez, and Chrostowski, 2004), fourth- and eighth-grade students from Singapore achieved the top average scores in mathematics. This paper compares the Singapore and U.S. primary mathematics system to explore what the United States can learn from the Singapore system of elementary mathematics education that may help improve the mathematics performance of U.S. students. It also identifies a few areas where the U.S. mathematics system may be preferred to Singapore's system.

Before proceeding with the Singapore-United States comparisons, we need to address three misimpressions about the Singapore primary mathematics system. First, many experts (National Academy of Sciences, 2005) believe that U.S. primary students already perform well on international mathematics assessments and that the focus of U.S. mathematics reform should be on the upper grades. This common perception is based on published reports showing U.S. students scoring above the international average on grade 4 TIMSS, but falling considerably below the average at age 15 on the OECD Program for International Student Assessment (PISA; NCES, 2005). But many industrialized European countries that outscore the United States on PISA do not participate in TIMSS. When the United States is compared against the 12 industrialized countries that participate in both TIMSS and PISA, its rank is consistently mediocre on both assessments — 8th on grade 4 TIMSS and 9th on grade 8 TIMSS and PISA, age 15. In general, countries that score well at grade 4 also do so at age 15, so establishing a sound foundation in early mathematics concepts is critical for later learning of more complicated mathematics concepts (Ginsburg, Cooke, Leinwand, Noell, and Pollock, 2005).

Second, it is argued that Singapore's student population is not as diverse as the U.S. student population. But Singapore's population is far from homogenous; almost a quarter of its students are Malay or Indian, along with its majority Chinese population. Further, Singapore's minority students far outperform U.S. minority students. To illustrate, four out of five Singapore's Malaysian students score in the top half of the TIMSS-2003 at

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grade 8, whereas a majority of U.S. black students score in the bottom quarter of the TIMSS international student rankings (Ministry of Education (MOE), Singapore, 2000; NCES, 2000).

Third, it is argued that Singapore's small population of about 4 million allows the development of a strong centralized curriculum not possible to replicate in the highly decentralized U.S. education system. True, but U.S. states under the federal No Child Left Behind legislation must have in place challenging mathematics standards and uniform assessments in at least grades 3–8, and states could adopt a curriculum like Singapore's (U.S. Congress, 2002).

Given Singapore's success, it is worthwhile to carefully analyze Singapore's mathematics education system and how it compares with the U.S. education system as the United States seeks to achieve the goals and mandates of No Child Left Behind. The comparisons of the two systems are made in the areas of frameworks, textbooks, assessments, and teachers, with the key findings described below.

Frameworks

The United States lacks Singapore's uniform and coherent national mathematics curriculum that guides its mathematics program.

Singapore has a well-defined, specific national mathematics framework. By contrast, the United States has no official national mathematics framework. In fact, the U.S. Department of Education is prohibited by law from specifying a national curriculum (U.S. Congress, 1979). Instead, the National Council of Teachers of Mathematics (NCTM, 2000) standards are the closest U.S. approximation to national standards and are the basis for many state frameworks.

However, the NCTM framework lacks Singapore's highly logical and specific mathematics framework. Singapore specifies grade-specific content, but NCTM specifies imprecise content only within broad grade bands (K–2, 3–5). Singapore adopts a spiral approach that builds outward on prior content; the U.S. NCTM uses a spiral approach that to a significant degree repeats content across grades.

The treatment of multiplication in grades 1 and 2 illustrates the differences between the Singapore and NCTM framework approaches. Singapore's grade 1 standards are specific and bounded – multiplication of numbers with a product not greater than 40 and division of numbers with a result not greater than 20. Grade 2 expands multiplication and division within the 2, 3, 4, 5, and 10 times tables.

In contrast with Singapore's specific and cumulative approach, NCTM covers multiplication and division in grades 1 and 2 within a broad K–2 grade band and within the general category of the "meaning of operations" rather than multiplication and division. Students are to "understand situations that entail multiplication and division, such as equal groupings of objects and sharing equally." These NCTM statements are too broad and unspecific to provide the direction necessary to guide what mathematics states and school systems should teach at specific grades.

State frameworks differ greatly; some state frameworks are nearly as specific and focused as Singapore's but others are poorly organized and lack direction.

The U.S. state mathematics frameworks differ considerably in the amount and nature of the mathematics content at each grade. Table 1 illustrates the differences between the mathematics frameworks of seven states and that of Singapore. Three of the seven states, California, North Carolina, and Texas, expose students to nearly the same amount of content as Singapore. Florida, in contrast, specifies about twice the number of mathematics topics per grade; teaches each topic twice as long; and also expects students to learn twice the number of outcomes per grade. Maryland, New Jersey, and Ohio frameworks fall in between in content exposure. With so much more mathematics content to cover in each grade, many U.S. state frameworks are not able to treat topics with depth. Therefore, they repeat the same topics over many more grades than does Singapore, and the mathematics curriculum does not progress at as fast a pace.

Table 1. Comparison of Content Exposure for Singapore and Selected U.S. States Mathematics Standards: Grades 1–6

	Total No. of Topics	Avg. No. of Topics/Grade		Avg. No. of Grades/Topic		Total No. of Outcomes	Avg. No. of Outcomes/ Grade	
		No.	Ratio to Sing.	No.	Ratio to Sing.		No.	Ratio to Sing.
	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Singapore	40	15	—	2.3	—	232	39	—
California	42	20	1.3	2.9	1.3	305	51	1.3
Florida	54	39	2.6	4.2	1.8	640	107	2.7
Maryland	46	29	1.9	3.8	1.7	415	69	1.8
New Jersey	50	28	1.9	3.4	1.5	336	56	1.4
North Carolina	41	18	1.2	2.6	1 .1	217	36	.9
Ohio	48	26	1.7	3.3	1.4	370	62	1.6
Texas	40	19	1.3	2.8	1.2	265	44	1.1

Source: Ministry of Education (MOE), Singapore (2001); States are from standards available in 2005 from individual state websites.

The treatment of fractions in grade 4 in Singapore and in Florida illustrates Singapore's greater focus and depth of presentation. Singapore specifies that students should be able to do the following:

- Add and subtract like fractions; related fractions. (Denominators of given fractions should not exceed 12; Exclude sums involving more than 2 different denominators.)
- Calculate the product of a proper fraction and a whole number.
- Express an improper fraction as a mixed number, and vice versa (Include expressing an improper fraction/mixed number in its simplest form).
- Solve up to two-step word problems involving fractions. (Include using unitary method to find the "whole" given a fractional part. Exclude question such as "Express the number of girls as a fraction of the number of boys" as it will be dealt with under the topic "Ratio.")

Contrast Singapore's clear focus on the mathematics of fractions with Florida's grade 4 specifications involving fractions:

- Compares and orders commonly used fractions and decimals to hundredths using concrete materials, drawings, and numerals.
- Locates whole numbers, fractions, mixed numbers, and decimals on a number line.
- Translates problem situations into diagrams and models using whole numbers, fractions, mixed numbers, and decimals to hundredths including money notation.
- Uses concrete materials to model equivalent forms of whole numbers, fractions, and decimals.
- Knows that two numbers in different forms are equivalent or nonequivalent, using whole numbers, decimals, fractions, and mixed numbers.
- Explains and demonstrates the addition and subtraction of common fractions using concrete materials, drawings, story problems, and algorithms.
- Explains and demonstrates the addition and subtraction of decimals (to hundredths) using concrete materials, drawings, story problems, and algorithms.
- Predicts the relative size of solutions in: addition and subtraction of common fractions.
- Uses problem-solving strategies to determine the operation(s) needed to solve one- and two-step problems involving addition and subtraction of fractions.

Florida's framework mixes the learning of whole numbers, decimals, and mixed numbers with fractions. Florida's descriptions also intersperse what students need to know about fractions with very general prescriptions about teaching fractions using concrete materials and drawings. In addition, Florida's descriptions lack the precision of Singapore's statements, such as "calculate the product of a proper fraction and a whole number."

Singapore recognizes that some students may have more difficulty in mathematics and provides them with an alternative framework; the U.S. frameworks make no such provisions.

Singapore has developed an alternative mathematics framework for lower-performing students in the upper primary grades that covers all the mathematics topics in the regular framework, but at a slower pace and with greater repetition. Singapore also provides its slower students with extra help from well-trained teachers. NCTM and the states we examined provide no alternative framework for lower-performing mathematics students. Moreover, such students are often tracked into slower mathematics courses, but unlike in Singapore, these students are seldom taught all the required mathematics material. Evaluations have shown that they frequently receive their extra help from teacher's aides who lack college degrees and/or background in mathematics.

Table 2. Comparison of Textbook Organization by Number of Topics, Lesson Length, and Pages Devoted to Development and Exercises: Grade 3

	Total Pages	# of Chapters	# of Lessons	Average Pages per Lesson	Pages of Development	Pages of Exercises	Other Pages
Singapore	496	14	42	12	150 (30%)	234 (47%)	112 (23%)
Scott-Foresman	729	32	164	4	187 (26%)	217 (30%)	325 (45%)
Everyday Math	257	11	120	2	137 (53%)	120 (47%)	—

Source: Singapore Ministry of Education (2001), Scott-Foresman (2004), and Everyday Mathematics (2001).

Textbooks

Singapore textbooks' in-depth treatment of mathematical topics build deep understanding of mathematical concepts; traditional U.S. textbooks rarely get beyond definitions and formulas, developing primarily students' mechanical ability to apply mathematical concepts.

Singapore textbook publishers know the mathematics that students are expected to learn grade by grade. U.S. textbook publishers know the textbook content for some states with well-defined frameworks, but not for other states where frameworks specify mathematical content only in a general way. Even when states have well-defined mathematics topics in each grade, the same topics are taught over different grades in different states. As an example, students are expected to gain fluency in the addition of fractions by grade 4 in one state; grade 5 in 15 states; grade 6 in 20 states; and grade 7 in 6 states (Reys et al., 2005).

U.S. textbook publishers seek to maximize a textbook's market share across states with different frameworks by including lessons on many more topics in the same textbook than is the case in Singapore. To quantify this difference in textbook topic coverage, we compare the Singapore mathematics textbook and a traditional U.S. textbook we selected, the Scott-Foresman mathematics series. We also selected a nontraditional U.S. mathematics textbook, *Everyday Math,* which was developed as part of the National Science Foundation textbook development effort.

These three textbooks are compared in their treatment of the grade 3 topics in Table 2. Comparisons for other grades yield similar results. The Singapore textbook covers half the number of topics as the two U.S. textbooks. Moreover, there are 12-page lessons on average in Singapore, compared with Scott-Foresman's short

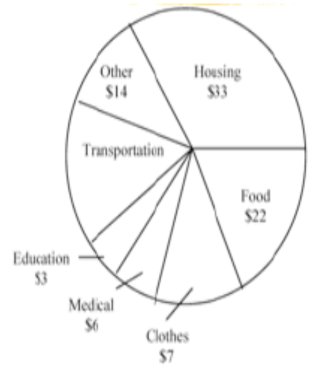
4-page lessons. Although the U.S. traditional textbook has almost double the pages of Singapore's textbook, about 45 percent of the pages do not cover either concept development or exercises, far more than for either the Singapore textbook or *Everyday Math*.

Exhibit 1a. Singapore's Pie Chart Problem With Intermediate Unknowns Involving Angles


The pie chart represents the amount of money collected by various stalls at a funfair.

(a) What fraction of the total amount of money was collected by the games stalls?
(b) What was the total amount of money collected by the various stalls?
(c) How much money was collected by the music stalls?
(d) What was the ratio of the money collected by the food stalls to the money collected by the handicraft stalls?

Exhibit 1b. Scott-Forseman Pie Chart Problem With Mechanical Application of Definition of Pie Chart.



(a) What is the cost of transportation?
(b) For each $100 a parent spends raising a child to age 18, how much more is spent on housing and clothes than on education?
(c) TEST PREP. For each $300 spent, estimate how much is spent for food and clothes
1) $329 2) $90 3)$29 4)$130
(d) Which costs are about twice as much as the cost of education? Five times as much? Eleven times as much?

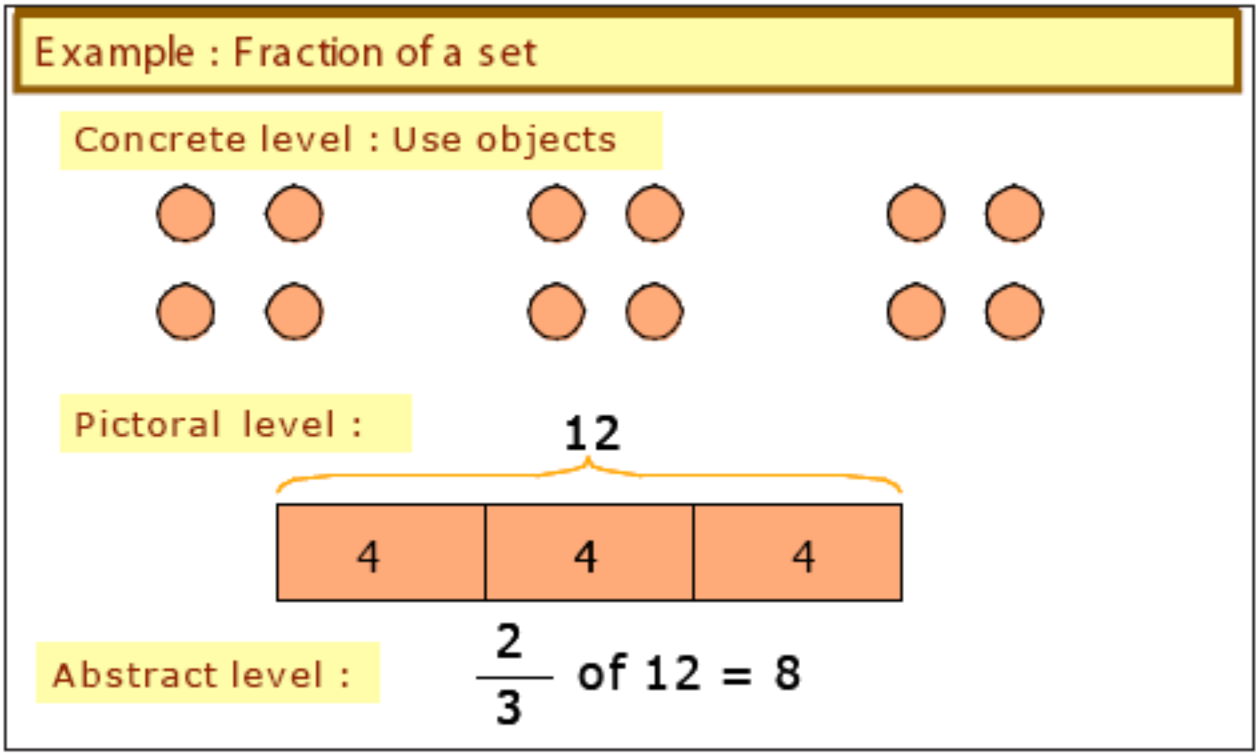
The lengthier lessons allow the Singapore textbook to have rich and cumulative topic development. We illustrate the richness of the Singapore development with the pie chart problem in Exhibit 1a. Students need to understand that the size of the pie slices are in proportion to the size of the angles in a circle and draw on their knowledge of right angles and the sum of angles of a straight line to solve the problem. The pie chart example from the Scott-Foresman textbook (Exhibit 1b) does not address angle measures and requires knowing only that the sum of the pieces of the pie have to equal the total, or 100 percent.

Singapore's Concrete-Pictorial-Abstract Approach

Students begin to grasp the power of mathematics when they are able to apply the essential core mathematical principles to solve a rich variety of mathematics problems. But to grasp mathematical ideas, students must understand and apply mathematical abstractions, yet abstract mathematics is difficult for many students to grasp. The Singapore textbook features instructional presentations in which a concept is first illustrated concretely, then pictorially, and finally abstractly. The approach is similar in purpose to the manipulatives common in U.S. schools, but Singapore tightly connects its concrete examples with student learning of mathematical ideas, a frequent weakness with the U.S. application of manipulatives.

Exhibit 2 illustrates the concrete-pictorial-abstract approach applied to explaining how dividing 12 by 3 is the same as multiplying 12 by one-third. The concrete level displays thirds in terms of objects. The pictorial level shows thirds represented by a bar. Finally, the abstract level expresses the fraction of thirds in terms of an equation. Singapore also builds up the pictorial approach to show how to use diagrams to translate complicated word problems into mathematical expressions. Similar consistent presentations of the concrete, pictorial, and abstract approach were not found in the traditional U.S. mathematics textbooks.

Exhibit 2. Singapore's Concrete-Pictorial-Abstract Approach Provides a Bridge for Students Who Have Trouble With Abstract Mathematics Concepts



Source: Singapore Primary Math Grade 3

Assessment

Singapore has uniform assessments at two transition grades, 6 and 10; the United States mandates testing in more grades, but the 50 state assessments of mathematics differ significantly in how they measure student performance.

Singapore's uniform assessments are developed by the Ministry of Education at grades 6 and 10. These are high stakes placement exams. The uniform national examination covers the cumulative mathematics content in Singapore's mathematics framework. In the United States, there is no official national assessment. The NCLB Act does provide for individual state assessments in grades 3–8 and one high school grade. At the national level, the U.S. Department of Education administers the National Assessment of Educational Progress (NAEP), but it is given to only a sample of each state's students in mathematics in grades 4 and 8; scores are reported for states, but not for individual students.

In response to NCLB, each state's tests are variable yardsticks on which a student's test results often depend on the state in which he or she resides. One way to quantify the differences in state measures is to compare a state's pass rate of its students on its own test with its pass rate on the uniform national NAEP test for the same grade. South Carolina is a state that has set a stringent state mathematics test, on which only 23 percent of the state's students achieved proficient levels, below the 30 percent at least proficient on NAEP's mathematics assessment. Tennessee is a state that established much lower pass standards for students to achieve proficient or above on its own test. Although 87 percent of Tennessee's grade 8 students were proficient on Tennessee's own state test, only 21 percent were proficient on NAEP (Dillion, 2005). Until the United States decides to have state tests match up with a uniform national test like NAEP, these disparities in test measures across states will continue.

Singapore's school accountability approach measures the school's progress of the same students over grades (i.e., value-added); the U.S. school accountability approach in NCLB measures the progress of different students at the same grade over time.

Singapore uses a value-added measure of learning growth. Grade 10 actual scores are compared with grade 10 predicted scores, where predicted scores are based on grade 6 student scores. Those schools with actual grade 10 scores above predicted grade 10 scores are rewarded with recognition and funding (Tan, 1995).

NCLB measures school performance by whether school outcomes at each grade demonstrate adequate yearly progress (AYP) with successive cohorts of students at the same grade. States establish annual improvement goals for each grade, and schools must meet state progress at each grade. Schools need to continue to demonstrate progress so that by 2014, all students in a school achieve proficient levels. A criticism of this approach is that schools that have more needy students and that start with lower performing students must demonstrate more growth to reach proficiency than schools with higher performing students. Hence, schools are discouraged from taking low-income students, for example, where school choice plans exist. This is not a problem with Singapore's value-added approach in which schools are compared by the amount of improvement. To test the feasibility and benefits of a value-added measure, Secretary of Education Spellings recently announced a pilot that would allow some states to use a Singapore-like, value-added approach on a trial basis (U.S. Department of Education, 2005).

The questions on Singapore's high stakes grade 6 Primary School Leaving Examination (PSLE) are more challenging than the released items on the U.S. grade 8 National Assessment of Educational Progress (NAEP) or the grade 8 state assessments.

The items on the Singapore assessments are more likely to require deeper problem solving skills than items on the U.S. state assessments or NAEP at grade 8. Items on Singapore's tests are more likely to require preparing constructed responses, solving for intermediate unknowns, and developing non-routine solutions than are U.S. test items (see Table 3).

Table 3. Comparison of Assessment Items for Singapore, Selected States, and NAEP
(Percent of Items)

	Multiple Choice	Multistep	Finding an Intermediate Unknown
Singapore – Gr. 6	31%	25%	19%
Florida – Gr. 8	52%	12%	8%
New Jersey – Gr. 8	85%	33%	0%
N. Carolina – Gr. 8	100%	5%	5%
Ohio – Gr. 6	74%	17%	4%
Texas – Gr. 8	100%	6%	2%
NAEP* - Gr. 8	65%	21%	8%

Source: State data based on released items; NAEP data based on entire item pool.

Teachers

Singapore teachers must pass a rigorous entrance exam to be accepted to education school, which they are paid to attend; U.S. teachers most frequently pass the PRAXIS I and PRAXIS II elementary teacher exams, which are no harder than the Singapore grade 6 exam.

Singapore targets acceptance to education school from the top third of its graduates by using a rigorous exam as part of the screening process. Singapore's primary education teachers typically have only two years of a college education, but during this period they take 12 semester hours of mathematics pedagogy courses covering the content and teaching of all the major topics in the Singapore framework.

U.S. elementary teachers are among the poorest performers of all college students in mathematics, as measured by their SAT scores, which are significantly below the SAT scores of the average college graduate (Gitomer, Latham, and Ziomek, 1999). Education majors take less mathematics than the typical college graduate, 6.3 credit hours compared with 8.3 credit hours (NCES, 2002). They are able to start with low mathematics scores and take few courses to improve their mathematics because of the low requirements of the PRAXIS I at the beginning of education school and the PRAXIS II elementary exams taken for state licensing.

The most common exams that U.S. teachers have to pass are the PRAXIS I to graduate from education school and the PRAXIS II to obtain a state license. As the typical PRAXIS I questions illustrate, the PRAXIS elementary teacher mathematics questions are all multiple-choice questions, which rarely require examinees to do more than solve one-step, straightforward applications of a definition or concept (see Table 3). Contrast these questions with three illustrative questions at the easy or moderate level from Singapore's Primary School Leaving Exam at grade 6 (see Exhibit 3). The Singapore Ministry of Education designated easy questions as ones that all students would be expected to answer correctly and moderate questions as ones that average students would be expected to answer correctly. The Singapore questions require students to undertake multiple step solutions and demonstrate an understanding of concepts in nonroutine situations. Toughening the assessments that elementary teachers pass is one of the surest way to obtain teachers who understand core mathematics ideas.

Exhibit 3. Typical Questions on U.S. PRAXIS Exam for Prospective Elementary Teachers Compared With Singapore Grade 6 Primary School Leaving Exam Sample Problems (Easy or Moderate Difficulty)

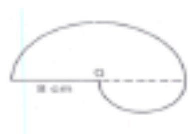
Typical U.S. PRAXIS I Teacher Elementary Exam Questions (source: http://ftp.ets.org/pub/tandl/0730.pdf))

1. Which of the following is equal to a quarter of a million?
   (A) 40,000 (B) 250,000 (C) 2,500,000 (D) 1/4,000,000 (E) 4/1,000,000
2. Which of the following fractions is least?
   (A) 11/10 (B) 99/100 (C) 25/24 (D) 3/2 (E) 501/500
3. Which of the sales commissions shown below is greatest?
   (A) 1% of $1,000 (B) 10% of $200 (C) 12.5% of $100 (D) 15% of $100 (E) 25% of $40
4. For a certain board game, two dice are thrown to determine the number of spaces to move. One player throws the two dice and the same number comes up on each of the dice. What is the probability that the sum of the two numbers is 9?
   (A) 0 (B) 1/6 (C) 2/9 (D) 1/2 (E) 1/5

Typical Singapore Grade 6 Primary School Leaving Exam

1. Singapore Grade 6 (easy) Question: Global pours an equal amount of water into two empty P and Q tanks shown below.

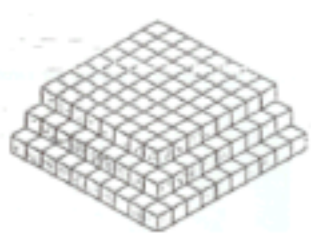
If tank P is half-filled, what is the height of the water level in Tank Q?
A) 5 cm B) 6 cm C) 8 cm D) 10 cm



2. Singapore Grade 6 (moderate). This figure is made up of two semicircles. Q is the centre of the larger semicircle of radius 8 cm.

The area of the figure is ______ cm^2.
(1) 12π (2) 40π (3) 48π (4) 80π

Singapore 6th grade (moderate). Jim uses 1-cm cubes to build a solid. He starts with a square base using 100 cubes.



The second layer of 81 cubes and the third layer of 64 cubes are also arranged in the form of squares. The figure shows the three layers of cubes.

Jim continues building the solid in the same way.

a) How many cubes will there be in the next layer?
b) If the topmost layer consists of 9 cubes, how many layers does the solid have?

Areas of Strengths in the U.S. Mathematics System Compared With Singapore's System

Although the U.S. mathematics program is weaker than Singapore's in most respects, the U.S. system is stronger than Singapore's in some areas.

The U.S. framework gives greater emphasis than Singapore's framework does to developing important 21st-century mathematical skills such as representation, reasoning, making connections, and communication.

Singapore's framework emphasizes a very traditional set of school processes. These include understanding concepts; performing processes including arithmetic; and strategic problem solving, such as heuristic strategies for formulating problems (e.g., use a diagram or model, work backward, simplify the problem, look for patterns).

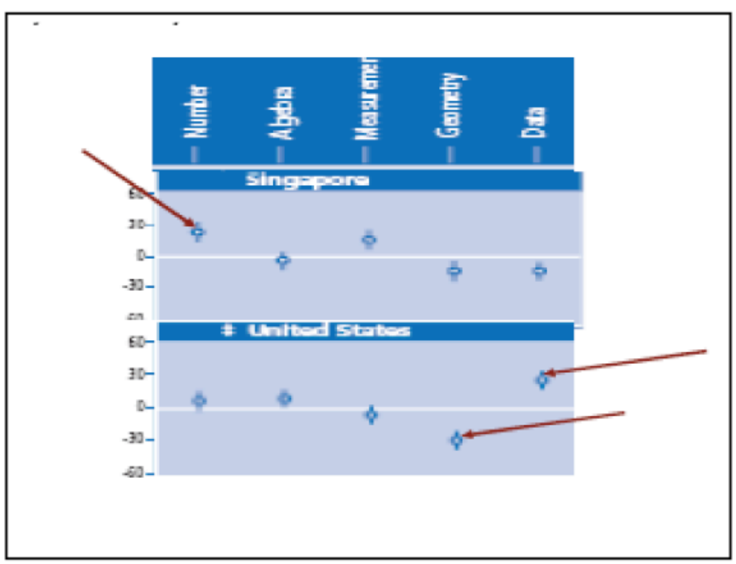
The U.S. NCTM and many state frameworks emphasize 21st-century skills rather than the traditional skills of Singapore's focus. Whereas Singapore has a single "process" category for strategic problem-solving skills, NCTM (2000) has three: reasoning and proof, representation, and connections, in addition to problem solving. NCTM makes communication of mathematical ideas a priority, but Singapore does not give it much emphasis and includes communication only as one on a list of skills. If U.S. students could grasp 21st-century mathematics skills, they could leap frog their international performance, but the NCTM and the state frameworks need to do a better job of integrating them with rigorous mathematics content.

The United States places a greater emphasis on applied mathematics, including statistics, probability, and real-world problem analysis.

The U.S. mathematics frameworks stress data analysis and probability, whereas the Singapore framework treats statistics in a strictly theoretical way. This difference in content emphasis is apparent in comparing the traditional U.S. textbook, which at grade 6 has 11 percent of the lessons about data and statistics, with Singapore's textbook, which has only 1 percent. Exhibit 4 shows that the relative time devoted to a standard makes a difference as evidenced by the fact that the U.S. scores on TIMSS-8 are relatively strong in data (including statistics) and Singapore's scores are relatively weak in data.

The U.S. frameworks and textbooks also place greater emphasis on mathematical

Exhibit 4. Relative Strengths and Weaknesses of Singapore and U.S. Curriculum, TIMSS-8, 2003.



Source: Mullis, I., Martin, M., Gonzalez, E., & Chrostowski, S. (2004)

applications. This is clearly seen in the nontraditional textbook, *Everyday Math*, which uses a problem based learning approach, often involving multi-step, real-world mathematics problems as the primary method of instruction. Such applications give students practice in understanding how to apply mathematics in practical ways. Singapore's textbooks would benefit from more real world applications.

Implications

Singapore's mathematics system is far better designed to truly *leave no child behind* than is the current U.S. mathematics system. Singapore's framework meets NCLB provisions to set "challenging academic standards." Its instruction is "research based." Its teachers are truly "highly qualified," as measured by their knowledge of mathematics, and its assessments are "aligned" with the content frameworks and assess students' mathematics knowledge at both "proficient and advanced levels." The result is that Singapore students, including its minority students, are among the best students in the world. The United States, while retaining its emphasis on statistics, applications, and 21st century skills, would do well to model its mathematics reforms after the superior features of Singapore mathematics.

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